Solution to Assignment 4

15.5

(24). The region is over a rectangle which can be decomposed into two trianlges D_1 and D_2. D_1 has vertices at $(0,0),(1,0),(1,2)$ and D_2 has vertices at $(0,0),(1,2),(0,2)$. Over D_1, the region is described by $0 \le z \le 1-x$. Over D_2, it is given by $0 \le z \le (2-y)/2$. Hence the volume of the region is

$$\iint_{D_1} \int_0^{1-x} 1\, dz\, dA(x,y) + \iint_{D_2} \int_0^{(2-y)/2} 1\, dz\, dA(x,y) \ = \cdots \ .$$

(27). Let $\mathbf{v}_1 = (0,2,0)-(1,0,0) = (-1,2,0)$ and $\mathbf{v}_2 = (0,0,3)-(1,0,0) = (-1,0,3)$. Then $(a,b,c) = \mathbf{v}_1 \times \mathbf{v}_2 = (6,3,2)$. The equation of the plane passing through $(1,0,0),(0,2,0),(0,0,3)$ is given by $6x+3y+2z = d$. Setting $(x,y,z) = (1,0,0)$, the equation is $6x+3y+2z = 6$. Regarding it as a region over the triangle T in the xy-plane with vertices at $(0,0),(1,0),(0,2)$, the volume of the tetrahedron is

$$\iint_T \int_0^{(6-6x-3y)/2} dz\, dA(x,y) = \cdots \ .$$

(29) The region is described by $0 \le z \le \sqrt{1-x^2}$ where (x,y) satisfies $x^2+y^2 \le 1, x,y \ge 0$. Therefore, the volume of this region is

$$8 \times \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2}} 1\, dz\, dydx = \cdots = 16/3 \ .$$

(42) Change the order of $dydx$ to $dxdy$:

$$\begin{aligned} \int_0^1 \int_{x^2}^1 12xze^{zy^2}\, dydx &= \int_0^1 \int_0^{\sqrt{y}} 12xze^{zy^2}\, dxdy \\ &= \int_0^1 6yze^{zy^2}\, dy \ . \end{aligned}$$

Hence

$$\begin{aligned} \int_0^1 \int_0^1 \int_{x^2}^1 12xze^{zy^2}\, dydxdz &= \int_0^1 \int_0^1 6yze^{zy^2}\, dydz \\ &= 3\int_0^1 e^{zy^2}\Big|_0^1\, dz \\ &= 3\int_0^1 (e^z-1)\, dz \\ &= 3(e-2) \ . \end{aligned}$$

Supplementary Problems

1. Find the equations of the planes passing through the origin and (a) $(1,2,3),(0,-2,0)$ and (b) $(0,2,-1),(3,0,5)$.

   Solution. (a) $(1,2,3)\times(0,-2,0) = (6,0,-2)$. The equation is $6x-2z=0$ or $3x-z=0$.

   (b) $(0,2,-1)\times(3,0,5) = (10,-3,-6)$. The equation is $10x-3y-6z=0$.

2. Find the equation of the plane passing the points $(1, 0, -1), (4, 0, 0), (6, 2, 1)$.

   Soluton. Take $\mathbf{u}_0 = (4, 0, 0)$. (You can take any one of these three points as the base point.) Then $\mathbf{v}_1 = (1, 0, -1) - (4, 0, 0) = (-3, 0, -1)$, and $\mathbf{v}_2 = (6, 2, 1) - (4, 0, 0) = (2, 2, 1)$. $\mathbf{v}_1 \times \mathbf{v}_2 = (2, 1, -6)$. The equation is $2x + y - 6z = d$. Since $(4, 0, 0)$ belongs to the plane, $d = 2 \times 4 + 0 - 6 \times 0 = 8$. The equation of this plane is $2x + y - 6z = 8$.